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ON THE PERFORMANCE OF A MODIFIED SIGN DETECTOR FOR M-DEPENDENT --ETC

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## ON THE PERFORMANCE OF A MODIFIED SIGN DETECTOR FOR M-DEPENDENT DATA

D.R. HALVERSON  
Department of Electrical Engineering  
Texas A&M University  
College Station, Texas 77843

and

G.L. WISE  
Department of Electrical Engineering  
University of Texas at Austin  
Austin, Texas 78712

### ABSTRACT

A nonparametric detection scheme, the modified sign detector, may be applied to the discrete time detection of a constant signal in additive m-dependent noise. It is shown how the optimal block size for this detector may be selected for two fidelity criteria, one based on a finite number of samples and the other on the asymptotic limit. These results may then be used to exhibit examples of cases where a small block length is indicated by the finite sample criterion, whereas a large block length is indicated by the asymptotic criterion.

### I. INTRODUCTION

The employment of a nonparametric detector is often desirable in situations where little information about the statistics of the noise is available. If the noise sequence is independent and identically distributed, a popular choice is the well known sign detector [1]. Because of modern high speed sampling, however, in many situations it is unlikely that adjacent samples of the waveform could be considered to be independent. What we might expect is that samples separated sufficiently far apart in time could be considered to be independent, i.e. an assumption of m-dependence is often reasonable. In these cases the sign detector unfortunately loses its nonparametric nature. It is thus desirable, when confronted with dependency in the noise, to modify standard nonparametric schemes in a way which is easily implemented and yet preserves the nonparametric nature of the detector under dependent inputs. One promising approach toward this goal has been considered in [2]. In this paper we investigate more thoroughly the performance of the method of [2] as applied to the sign detector, and also illustrate how this investigation provides insight into some fundamental questions pertaining to the general notion of optimality of a detector.

### II. DEVELOPMENT

Suppose the noise process  $\{X_i; i=1,2,\dots\}$  is stationary and m-dependent (i.e.  $k-j>m$  implies  $\{X_i; i=1,2,\dots,j\}$  and  $\{X_i; i=k,k+1,\dots\}$  are independent) with continuous diagonally symmetric joint densities of all orders. We will wish to decide between

$$H_0: Y_i = X_i \quad ; \quad i = 1, \dots, N_0$$

$$H_1: Y_i = X_i + s \quad ; \quad i = 1, \dots, N_0$$

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where we observe realizations  $\{y_i; i=1, \dots, N_0\}$  of the process  $\{Y_i; i=1, \dots, N_0\}$  and  $s$  is a known positive constant. As applied to the sign detector, the method of [2] requires grouping the samples into blocks of length  $n$  with  $m$  samples skipped between blocks, resulting in  $N_n = G[(N_0+m)/(n+m)]$  independent blocks, where  $G(\cdot)$  is the greatest integer function. In general, we will allow fewer than the maximal number  $N_n$  blocks and will denote the number of blocks by  $N$ . If a summing operation is performed on each block, we obtain a sequence of independent random variables  $Z_{1,n}, \dots, Z_{N,n}$  upon which the standard sign detector may be applied. Note that in this case the "modified signal", if present, has strength  $ns$ . We will call such a detector a modified sign detector.

A question which naturally arises for this detection scheme is what choice of block length  $n$  gives the best performance. The measure of the fidelity of a nonparametric detector has been traditionally vested in employment of the Asymptotic Relative Efficiency (ARE) criterion, which is generally held to be especially appropriate when the signal is weak or the number of observations  $N_0$  is large. When employing the ARE criterion, we often make use of an expression for the ARE given by the Pitman-Noether theorem [3]. In order to apply this theorem, we will need to consider several regularity conditions expressed as follows in terms of the test statistics  $T_N(Y_1, \dots, Y_N)$  which, for the application considered here, may be taken to be zero mean unit variance random variables under  $H_0$  for all  $N$ . In the following,  $E_s\{\cdot\}$  denotes expectation computed under  $H_1$  with signal strength  $s$  and  $K$  is a positive constant.

- A.  $\frac{\partial}{\partial s} E_s\{T_N(Y_1, \dots, Y_N)\} \big|_{s=0} > 0$
- B.  $\lim_{N \rightarrow \infty} \frac{1}{N} \left[ \frac{\partial}{\partial s} E_s\{T_N(Y_1, \dots, Y_N)\} \big|_{s=0} \right]^2 > 0$
- C.  $\lim_{N \rightarrow \infty} \frac{\frac{\partial}{\partial s} E_s\{T_N(Y_1, \dots, Y_N)\} \big|_{s=K/\sqrt{N}}}{\frac{\partial}{\partial s} E_s\{T_N(Y_1, \dots, Y_N)\} \big|_{s=0}} = 1$
- D.  $\lim_{N \rightarrow \infty} E_{K/\sqrt{N}}\{(T_N(Y_1, \dots, Y_N) - E_{K/\sqrt{N}}\{T_N(Y_1, \dots, Y_N)\})^2\} = 1.$

For a large class of noise processes, the problem of choosing the optimum block length  $n$  is addressed by the following:

**Theorem 1:** Suppose, for the test statistics (where here  $N = N_n$ )

$$T_{N,n} = \frac{\sum_{i=1}^N \text{sgn} Z_{i,n}^{-N/2}}{\sqrt{N/4}}, \text{ that conditions A, B, C, D above are satisfied}$$

for all fixed choices of block lengths  $n$  and some positive constant  $K$ . Then the modified sign detector which maximizes the ARE relative to any other such detector has block length  $n$  if and only if  $n$  maximizes the quantity  $\tilde{n} \cdot f_n(0)$ , where  $f_n(\cdot)$  is the univariate density of  $Z_{1,n} = X_1 + X_2 + \dots + X_n$  and  $\tilde{n} = n/\sqrt{n+m}$ .

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Proof: It follows from the central limit theorem that  $T_{N,n}$  converges in distribution to a zero mean unit variance Gaussian random variable as  $N \rightarrow \infty$ , and thus the hypothesis allows application of the Pitman-Noether theorem [3], with the result that an expression for the ARE is given in terms of the efficacy  $\eta(\cdot)$ . Choice of the optimal block length  $n$  thus amounts to choosing  $n$  so as to maximize

$$\eta(n) = \lim_{N_0 \rightarrow \infty} \left[ \frac{\frac{\partial}{\partial s} E_s \left\{ \frac{\sum_{i=1}^N \text{sgn } Z_{i,n}^{-N/2}}{\sqrt{N/4}} \right\} \Big|_{s=0}}{N_0 E_0} \right]^2 / \left\{ \frac{\sum_{i=1}^N \text{sgn } Z_{i,n}^{-N/2}}{\sqrt{N/4}} \right\}^2.$$

But  $\sum_{i=1}^N \text{sgn } Z_{i,n}$  is binomially distributed with parameters  $N$  and  $p_{ns} = \Pr\{Z_{1,n} > 0 | H_1\}$  and thus

$$\begin{aligned} \eta(n) &= \lim_{N_0 \rightarrow \infty} \frac{\left[ \frac{\partial}{\partial s} N p_{ns} \Big|_{s=0} \right]^2 \cdot \frac{4}{N}}{N_0 \cdot 1} = \left[ \frac{\partial}{\partial s} \int_0^\infty f_n(x - ns) dx \Big|_{s=0} \right]^2 \cdot 4 / (n+m) \\ &= [\tilde{n}f_n(0)]^2 \cdot 4. \end{aligned}$$

Maximizing  $\eta(n)$  thus is equivalent to maximizing  $\tilde{n}f_n(0)$ .

QED

If the noise is Gaussian with the "triangular" autocorrelation function, the following corollary to Theorem 1 provides more specific information:

Corollary: Suppose the noise is zero mean Gaussian with autocorrelation

$$R_i = \begin{cases} \frac{m+1-i}{m+1} \sigma^2, & |i| \leq m \\ 0, & |i| > m \end{cases}.$$

Then  $\tilde{n}f_n(0)$  increases strictly monotonically for  $n \geq 2m$ , and thus the optimal block length, as measured by the ARE, is obtained for  $n \geq 2m$  by taking  $n$  as large as possible.

Proof: It is straightforward to show that the hypothesis of Theorem 1 is satisfied for this noise process. Note that the assumptions on the admissible noise densities imply that the noise is zero mean. Letting

$\sigma_n^2$  denote the variance of  $Z_{1,n}$ , we have

$$\begin{aligned} \sigma_n^2 &= n R_0 + 2\{(n-1)R_1 + (n-2)R_2 + \dots + (n-m)R_m\} \\ &= \left[ n(m+1) - 2 \frac{1 \cdot m + 2(m-1) + \dots + m \cdot 1}{m+1} \right] \sigma^2. \end{aligned}$$

We then note that  $\tilde{n}f_n(0)$  is proportional to  $\frac{n}{\sigma_n \sqrt{n+m}}$ , and hence would need

$$\text{to show } h(n) \stackrel{\Delta}{=} \frac{n^2}{(n+m) \left[ n(m+1) - 2 \frac{1 \cdot m + 2(m-1) + \dots + m \cdot 1}{m+1} \right]}$$

is strictly increasing on  $\{n: n \geq 2m\}$ , i.e.  $\tilde{h}(n) \triangleq n^2/(n^2 + an - b)$  is strictly increasing on  $\{n: n \geq 2m\}$ , where

$$a = m-2 \frac{1 \cdot m + 2(m-1) + \dots + m \cdot 1}{(m+1)^2} \quad \text{and} \quad b = 2m \frac{1 \cdot m + 2(m-1) + \dots + m \cdot 1}{(m+1)^2}.$$

A function of the form of  $\tilde{h}$  may be seen to possess a positive derivative for  $n > 2b/a$  (it is routine to show  $a > 0$ ). It thus suffices to show

$$n \geq \frac{4m(1 \cdot m + 2(m-1) + \dots + m \cdot 1)}{m(m+1)^2 - 2(1 \cdot m + 2(m-1) + \dots + m \cdot 1)}, \quad \text{and hence since } n \geq 2m, \text{ we must show}$$

$$1 \geq \frac{2(1 \cdot m + 2(m-1) + \dots + m \cdot 1)}{m(m+1)^2 - 2(1 \cdot m + 2(m-1) + \dots + m \cdot 1)}.$$

This may be seen to follow from an induction argument on  $m$ .

QED

As a consequence of these results, we would expect that in many cases best performance, as measured by the ARE, can be achieved by employing as large a block size as possible. In the next section we will see how, from a more pragmatic viewpoint, the opposite conclusion may be drawn. The resolution of this apparent disparity will turn out to lie in some limitations associated with the employment of an ARE fidelity criterion.

### III. THE FINITE SAMPLE CASE

We have seen that in many cases employment of the ARE fidelity criterion, as applied to the modified sign detector, leads to a choice of block length  $n$  as large as possible. From the engineering viewpoint, because of the necessity of utilizing only a finite amount of data, the preferred measure of fidelity would be relative efficiency and not the ARE. The chief reason for employing the ARE is that such an approach is generally more tractable than one based on relative efficiencies. It is generally assumed that if the number of samples is large, whatever can be said about the fidelity of a detector as judged by the ARE also applies if one elects to employ relative efficiency as the measure of fidelity. This is certainly true if the number of samples is sufficiently large, but there is a difficulty in determining what number of samples is sufficient, especially when the true noise statistics are known imperfectly. Because the ARE is a common choice of fidelity criterion for nonparametric detectors, the question of whether or not the ARE consistently reflects the measure of fidelity inherent in the relative efficiency criterion is of genuine concern.

In many cases, we would like to have the design of the detector not depend on a preassigned value of the detection probability  $\beta$ . One way to accomplish this would be to approach the measure of fidelity of the detector through a method which, for a finite number of samples, chooses the block length  $n$  so as to maximize  $\beta$  for small signals. This will also turn out to maximize the relative efficiency for certain values of  $\beta$  and signals  $s$ . The following theorem will show how this may be accomplished, and will allow addressing the question raised in the previous paragraph.

**Theorem 2:** Let  $\alpha$  be a fixed false alarm probability, and let  $r_N$  and  $t_N$  be the respective randomization probability and integral threshold required to achieve  $\alpha$  for the test statistic

$$\sum_{i=1}^N \text{sgn } z_{i,n}. \text{ Then } \left. \frac{d\beta}{ds} \right|_{s=0} = \frac{N n f_n(0)}{2^{N-1}} \left[ (1-r_N) \binom{N-1}{t_N} + r_N \binom{N-1}{t_N-1} \right]$$

$$\text{if } t_N \leq N-1, \text{ and } \left. \frac{d\beta}{ds} \right|_{s=0} = \frac{N n f_n(0) r_N}{2^{N-1}} \text{ otherwise.}$$

**Proof:** Note that  $t_N$  and  $r_N$  do not in fact depend on  $n$  since  $\sum_{i=1}^N \text{sgn } z_{i,n}$  is binomially distributed under  $H_0$  independently of  $n$ . We have, if  $t_N \leq N-1$ ,

$$\beta = \left[ \sum_{i=t_N+1}^N \binom{N}{i} p_{ns}^i (1-p_{ns})^{N-i} + r_N \binom{N}{t_N} p_{ns}^{t_N} (1-p_{ns})^{N-t_N} \right]$$

where  $p_{ns} = \int_0^\infty f_n(x-s) dx$ . Thus if  $t_N \leq N-1$ ,

$$\begin{aligned} \left. \frac{d\beta}{ds} \right|_{s=0} &= \left. \frac{d\beta}{dp_{ns}} \right|_{p_{ns}=\frac{1}{2}} \cdot \left. \frac{dp_{ns}}{ds} \right|_{s=0} \\ &= \left[ \sum_{i=t_N+1}^N \binom{N}{i} (i-(N-i)) \left(\frac{1}{2}\right)^{N-1} + r_N \binom{N}{t_N} (t_N-(N-t_N)) \left(\frac{1}{2}\right)^{N-1} \right] \cdot n f_n(0) \\ &= \frac{N n f_n(0)}{2^{N-1}} \left[ \sum_{i=t_N+1}^N \binom{N-1}{i-1} - \sum_{i=t_N+1}^{N-1} \binom{N-1}{i} + r_N \left\{ \binom{N-1}{t_N-1} - \binom{N-1}{t_N} \right\} \right] \\ &= \frac{N n f_n(0)}{2^{N-1}} \left[ (1-r_N) \binom{N-1}{t_N} + r_N \binom{N-1}{t_N-1} \right]. \end{aligned}$$

The desired result, if  $t_N \leq N-1$ , thus follows. If  $t_N = N$  an approach similar to the above completes the proof.

QED

The following lemma will simplify application of Theorem 2:

**Lemma 1:** For fixed block length  $n$  and false alarm probability  $\alpha$ , let  $\beta_1$  be the detection probability obtained from employing  $N_1$  blocks, and  $\beta_2$  from employing  $N_2$  blocks, where  $N_1 > N_2$ .

$$\text{Then } \left. \frac{d\beta_1}{ds} \right|_{s=0} \geq \left. \frac{d\beta_2}{ds} \right|_{s=0}.$$

**Proof:** Since  $n$  is fixed, this is equivalent to showing that the classical sign detector for  $N_1$  and  $N_2$  pieces of data respectively has this property.

Notice that

$$\left. \frac{d\beta_1}{ds} \right|_{s=0} - \left. \frac{d\beta_2}{ds} \right|_{s=0} = \left( \left. \frac{d\beta_1}{dp_{ns}} \right|_{p_{ns}=\frac{1}{2}} - \left. \frac{d\beta_2}{dp_{ns}} \right|_{p_{ns}=\frac{1}{2}} \right) n f_n(0) .$$

Since the sign detector is the locally optimal detector for independent stationary Laplace noise, we see that in this situation,

$$\left. \frac{d\beta_1}{dp_{ns}} \right|_{p_{ns}=\frac{1}{2}} \geq \left. \frac{d\beta_2}{dp_{ns}} \right|_{p_{ns}=\frac{1}{2}} ,$$

since the detector using  $N_2$  samples may be regarded as a detector using  $N_1$  samples (which ignores the extra samples). However, this inequality does not depend upon the assumption of Laplace noise, and we see that it holds in general.

QED

The importance of Lemma 1 is that it allows checking the relevant quantity given by Theorem 2 for fewer combinations of  $N$  and  $n$ . For each block length  $n$  we need only check the corresponding  $N_n = G[(N_0+m)/(n+m)]$ , which is the largest number of blocks permitted by the data. Note that because of the presence of  $f_n(0)$  in the quantity to be checked, we would not in general expect to be able to specify the optimal block length non-parametrically. It would be possible, for example, to estimate  $f_n(0)$  by the methods of [4] and [5], which provide pointwise estimates of a density. The great advantage here is that we only need to know the value of the density at the origin, and not elsewhere as in most parametric schemes. We remark that  $r_N$  and  $t_N$  do not depend on the underlying (symmetric) noise densities.

Suppose now that the noise is 10-dependent zero mean unit variance Gaussian with "triangular" autocorrelation

$$R_1 = \begin{cases} \frac{11-i}{11} , & |i| \leq 11 \\ 0 , & |i| > 11 \end{cases} ,$$

and suppose we have a large fixed number of samples  $N_0 = 300$ . We will, using Theorem 2 and Lemma 1 find the proper choice of block length  $n$  and corresponding  $N_n$  which maximizes  $\left. \frac{d\beta}{ds} \right|_{s=0}$ . To reduce the possibilities to be checked, we first need the following lemma.

**Lemma 2:** Suppose the noise is Gaussian with autocorrelation

$$R_1 = \begin{cases} \frac{m+1-i}{m+1} \sigma^2 , & |i| \leq m \\ 0 , & |i| > m \end{cases} .$$

For fixed number of blocks  $N$  and false alarm probability  $\alpha$ , let  $\beta_1$  be the probability obtained from employing a block length  $n_1$ , and  $\beta_2$  from a block length  $n_2$ , where  $n_1 > n_2 \geq m$ .



$$\text{Then } \left. \frac{d\beta_1}{ds} \right|_{s=0} > \left. \frac{d\beta_2}{ds} \right|_{s=0} .$$

Proof: From the proof of Theorem 2, it suffices to show  $n_1 f_{n_1}(0) > n_2 f_{n_2}(0)$ . This follows from methods similar to the proof of the corollary to Theorem 1.

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Using Lemma 1 and Lemma 2, it thus suffices to consider only those pairs of  $n$  and  $N$  given in the following table, where

$\left. \frac{d\beta}{ds} \right|_{s=0}$  is computed from Theorem 2 for  $(n, N)$ :

Table 1. Values of  $\left. \frac{d\beta}{ds} \right|_{s=0}$  for Gaussian noise with "triangular" autocorrelation

$(n, N)$	$\left. \frac{d\beta}{ds} \right _{s=0}$
(300, 1)	.210
(145, 2)	.293
(93, 3)	.354
(67, 4)	.405
(52, 5)	.382
(41, 6)	.374
(34, 7)	.394
(28, 8)	.380
(24, 9)	.375
(21, 10)	.393
(18, 11)	.374
(15, 12)	.370
(13, 13)	.376
(12, 14)	.372
(10, 15)	.375
(9, 16)	.376
(8, 17)	.379
(7, 18)	.391
(6, 19)	.385
(5, 20)	.394
(4, 22)	.402
(3, 23)	.409
(2, 25)	.419
(1, 28)	.431

Note that for this example, with 300 samples, the highest detection probability for weak signals is achieved with the smallest block length ( $n_1 = 1$ ), and the lowest detection probability for the largest block length ( $n_2 = 300$ ). However, if one were using the ARE as a measure of fidelity, a choice of large block length ( $n \rightarrow 300$ ) would be indicated by the corollary to Theorem 1. Table 1 thus provides examples of cases where the relative fidelity between two detectors is not preserved as one passes from a finite sample situation to the asymptotic limit. It is likely that one would want to include Gaussian densities, or small perturbations of them, in the nonparametric class considered, and in this case we would know that within this class there exists a nonempty family of densities which exhibit a reversal in relative fidelity as one passes to the asymptotic limit. It is quite common practice to employ the ARE fidelity criterion without attention to the question of how many samples are required to insure that a reversal does not occur. The existence of such a family calls into question the appropriateness of that practice.

Similar difficulties arise when the problem is approached from the viewpoint of relative efficiency. The previous example guarantees that there exist two detectors  $D_1$  (with  $n_1=20$ ) and  $D_2$  (with  $n_2=300$ ) such that for a fixed false alarm probability  $\alpha$ , we have  $\beta_1 > \beta_2$  for sufficiently small  $s$ . If we let  $\beta = \beta_1$ , then at most 300 samples are required to achieve  $(\alpha, \beta)$  with  $D_1$ , but since  $\beta_2 < \beta$ , Lemma 1 implies that more than 300 samples are required for  $D_2$ . Thus the relative efficiency  $RE(D_1, D_2; \alpha, \beta, s) > 1$ . But  $n_1 < n_2$ , so the Pitman-Noether theorem, together with the proof of Theorem 1 and its corollary, guarantee that  $ARE(D_1, D_2) < 1$ .

The relative efficiency curve  $RE(D_1, D_2; \alpha, \beta, s)$ , regarded as a function of  $s$ , thus lies above unity for certain values of  $s$  before finally crossing unity to converge to the ARE below. Note that the point at which the curve crosses unity corresponds to a reversal in relative fidelity between the two detectors. From the remarks earlier in this paragraph, it is known that at least 300 samples are required before this crossover occurs.

We thus have nonpathological examples of cases for which one might be led to a choice of a large block length  $n$  if the ARE criterion is employed, but a small block length if the situation is viewed from the finite sample perspective. This is further evidence to suggest that the difference between relative efficiency and its asymptotic limit is more than simply that of a mathematical definition, and should be considered to be of genuine engineering concern. These results raise questions concerning the appropriateness of universal application of the ARE fidelity criterion as applied to nonparametric detection in dependent noise. Because one must work with a finite amount of data, it would appear that any indication of detector fidelity obtained through the ARE criterion must be interpreted with caution.

#### IV. CONCLUSION

We have shown how the optimal block size for the nonparametric modified sign detector may be selected for two fidelity criteria, one based on a finite number of samples and the other on the asymptotic limit. We have found by way of example that it is possible for the two criteria to disagree radically on the optimal block size, and that this occurs even when what might be regarded as a large amount of data is available. Viewed in the large, we have produced examples of pairs of nonparametric

detectors ( $D_1, D_2$ ) for which  $D_1$  is the "best" and  $D_2$  the "worst" as judged from the asymptotic viewpoint, but with the opposite situation when judged from the finite sample viewpoint. It would thus appear that cautious employment of the ARE criterion as applied to nonparametric detection in dependent noise would be indicated, with attention given to the minimal number of samples required to insure consistency between the two measures of fidelity whenever possible.

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